The Huxley Meeting On Analytic Number Theory

17 – 19 September 2014

School of Mathematics
Cardiff University

Location: The workshop will take place at the following address:

School of Mathematics
Cardiff University
Senghennydd Road
Cardiff, Wales, UK
CF24 4AG

All talks are in the room M/0.40. Coffee breaks and lunches are in the room M/1.02.

Programme:

Wednesday 17th September

9:30-10:10: Welcome coffee

10:10-12:15: Session I

10:10-10:15 Opening
10:15-11:15 Ben Green: Large gaps between primes
11:15-12:15 Trevor Wooley: Exponential sums associated with approximately translation-invariant systems

12:15-14:00: Lunch

14-15:00: Session II

14:00-15:00 James Maynard: Large gaps between primes

15:00-15:30: Coffee break

15:30-17:00: Session III

15:30-16:30 Antal Balog: Sums of dilates of a finite set
16:30-17:00 Nigel Watt: On the distribution of Gaussian primes

Thursday 18th September

9:45-10:15: Morning coffee

10:15-12:15: Session I

10:15-11:15 Roger Heath-Brown: Triples of almost-primes
11:15-12:15 Nina Snaith: Elliptic curves and random matrices

12:15-14:00: Lunch

14:00-15:00: Session II

14:00-15:00 Mark Coffey: Mellin transforms with factors with zeros only on the critical line

15:00-15:30: Coffee break

15:30-17:00: Session III

15:30-16:30 Christopher Hooley: On numbers that are the sum of a prime and two square free squares
16:30-17:00 Jose Antonio Adell: Differential calculus for linear operators and its applications to analytic number theory

19:00: Workshop dinner in Aberdare Hall.

Friday 19th September

9:45-10:15: Morning coffee

10:15-12:15: Session I

10:15-11:15 Jens Marklof: Directions in hyperbolic lattices
11:15-12:15 Martin Huxley: A sideways approach to the circle problem

12:15-13:15: Lunch

13:15-14:15: Session II

13:15-13:45 Igor Wigman : On probability measures arising from lattice points on circles
13:45-14:15 Efthymios Sofos : Rational points on the Fermat cubic surface

14:15-14:45: Goodbye coffee

Abstracts

Differential calculus for linear operators and its applications to analytical number theory

José A. Adell

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Faculty of Science, University of Zaragoza

In the first part of the talk, we sketch a differential calculus for linear operators represented by a family of finite signed measures, in particular, by stochastic processes. Such a calculus is based on the notions of g-derived operators and processes and g-integrating measures, g being a right-continuous nondecreasing function. Depending on the choice of g, this differential calculus works for non-smooth functions and under weak integrability conditions. Various illustrative examples, involving the negative binomial and the gamma processes, are considered.

The second, and main part of the talk, is concerned with applications to analytical number theory, mainly referring to fast computations, by means of series with a geometric rate of convergence, of well known functions and constants, such as the logarithm, the inverse tangent, the Catalan constant, and the alternating zeta function, among others. Special attention is devoted to the computation of the sequence $(\gamma_n(a))_{n\geq 0}$ of the Stieltjes constants appearing in the Laurent expansion of the Hurwitz and Riemann zeta functions. We provide different approximations of each constant $\gamma_n(a)$ by means of finite sums or series involving Bernoulli numbers. The particular case of the Euler-Mascheroni constant is discussed in more detail.

Sums of dilates of a finite set

Antal Balog

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Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences

In this talk we discuss problems and results in the following form. For a fixed q the set A+qA must be big compared to A. For example, when q is a non zero integer and A is a finite set of integers, we have $|A + qA| > (|q| + 1)|A| - c$ with a constant depending on q alone. Here $|q| + 1$ is the best possible multiplicative constant, however, the best value of the additive constant $c = c(q)$ is not known. According to S. Konyagin and I. Laba, when q is a transcendental number and A is a finite set of real numbers, the lower bound for the size of $A + qA$ is bigger than linear. The best result is not known here.

Mellin transforms with factors with zeros only on the critical line

Mark W. Coffey (with Matthew Lettington)
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Families of Mellin transforms are presented which yield polynomial factors with zeros only on the critical line Re $s = 1/2$. The polynomials are identified as certain hypergeometric functions, and connections with combinatorial geometry are discussed. The polynomials have the functional equation $p_n(s) = \epsilon p_n(1-s)$, $|\epsilon| = 1$, and satisfy a recurrence relation and a crucial difference equation. Directions for future research will be indicated. Joint work with Matthew Lettington, Cardiff University.

Large Gaps between primes

Ben Green
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Mathematical Institute, University of Oxford

I will discuss work with Ford, Konyagin and Tao establishing that the largest gap between primes $< X$ is bigger than Rankin's 1938 bound by a factor tending to infinity with X.

Triples of almost-primes

Roger Heath-Brown
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Mathematical Institute, University of Oxford

Chen's Theorem tells us that there are infinitely many primes p for which $p+2$ is a P_2 almost prime, and a standard weighted sieve shows that there are infinitely many p for which $(p+2)(p+6)$, for example, is a P_7. We show how Chen's method can be adapted to control the number of prime factors of $p+2$ here, proving that there are infinitely many primes p such that $p+2$ is a P_2 and $p+6$ is a P_r. Our provisional value of r is 75 - disappointingly large!

On numbers that are the sum of a prime and two square free squares

Christopher Hooley
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School of Mathematics, University of Bristol

We discuss the proof of our result on large numbers that are the sum of a prime and two squares of square free numbers.

A Sideways Approach to the Circle Problem

Martin Huxley
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School of Mathematics, Cardiff University

The average of the sum-of-two-squares function $r(n)$ can be read as counting integer points in a circle with centre at the origin. Moving the centre, or changing the shape, hasn't helped yet, but it leads to some interesting problems.

Directions in hyperbolic lattices

Jens Marklof (with Ilya Vinogradov)
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It is well known that the orbit of a lattice in hyperbolic n -space is uniformly distributed when projected radially onto the unit sphere. In the present work, we consider the fine-scale statistics of the projected lattice points, and express the limit distributions in terms of random hyperbolic lattices. This provides in particular a new perspective on recent results by Boca, Popa, and Zaharescu on 2-point correlations for the modular group, and by Kelmer and Kontorovich for general lattices in dimension n=2 .

Large gaps between primes

James Maynard
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Montreal and Oxford University

In 1938 Rankin showed that there are pairs of consecutive primes less than x which differ by at least a fixed multiple of $\log x \log_2 x \log_4 x/(log_3 x)^2$, where $\log_v$ denotes the v-fold logarithm. It was a famous problem of Erdos to improve this bound by an arbitrarily large constant (for x sufficiently large). We will discuss how recent progress on small gaps between primes allows us to answer this question.

Elliptic curves and random matrices

Nina Snaith
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There is much evidence to support the Katz-Sarnak philosophy in the case of a family of quadratic twists of an elliptic curve. That is, as the conductor, the parameter that orders the curves in the family, becomes large, the zeros of the associated L-functions behave statistically like the eigenvalues of matrices from the orthogonal group O(N). In 2006 Steven J. Miller produced numerical evidence that for finite conductor, statistics of zeros in these families of L-functions are not well modelled by the group O(N). In this talk we will investigate some statistics of zeros when we are far from the large-conductor limit.

Rational Points on the Fermat Cubic Surface

Efthymios Sofos
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We use conic bundle fibrations to provide a straightforward proof of the result of Slater and Swinnerton-Dyer that provides a lower bound that agrees with Manin's prediction for the number of rational points of bounded height on smooth cubic surfaces containing two rational skew lines. The Fermat cubic surface does not satisfy this assumption and we extend our methods to cover this case as well. As an application we provide a simple counterexample to Manin's conjecture over $\mathbb{Q}$:

On the distribution of Gaussian primes

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Let $\mathbb{P}[i]$ be the set of all prime elements of $\mathbb{Z}[i]$. For $R \in (1,\infty)$, $\theta \in (0,1)$ and $\alpha \in [0,\pi/2)$, put

$$\mathcal{D}_{\theta,R}(\alpha) = \{z \in \mathbb{C} : |z - Re^{i\alpha}| \leq R^{\theta}\}$$

and

$$E_{\theta,R}(\alpha) = 0 \text{ if } \mathbb{P}[i] \cap \mathcal{D}_{\theta,R}(\alpha) \neq \emptyset,\ 1 \text{ otherwise.}$$

The set of all $\theta \in (0,1)$ such that $\lim_{R\to\infty} \int_0^{\pi/2} E_{\theta,R}(\alpha)\mathrm{d}\alpha = 0$ forms a non-empty subinterval $\mathcal{I}$ of the real line. Indeed, it is a trivial corollary of a result of Harman, Kumchev and Lewis [Trans. Amer. Math. Soc. **356** No. 2 (2003), 599-620] that the number $\theta_0 = \inf \mathcal{I}$ lies in the interval $[0, 0.53]$, while $\sup \mathcal{I} = 1$. This talk will describe recent work of the speaker on upper bounds for θ_0, including the determination of a number $b < 1/2$ that satisfies $b \geq \theta_0$. This work makes use of the speaker's results on weighted fourth moments of Hecke zeta functions with groessencharacters [preprint, arXiv:1307.5333 [math.NT], 2013; to appear in Moscow J. of Combinatorics & Number Theory, **4**, Issue 2].

On probability measures arising from lattice points on circles

Igor Wigman
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Kings College London

This work is joint with Par Kurlberg. A circle, centered at the origin and with radius chosen so that it has non-empty intersection with the standard integer lattice, gives rise to a probability measure on the unit circle in a natural way. Such measures, and their weak limits, are said to be attainable from lattice points on circles.

We investigate the set of attainable measures and show that it contains all extreme points, in the sense of convex geometry, of the set of all probability measures that are invariant under some natural symmetries. Further, the set of attainable measures is closed under convolution, yet there exist symmetric probability measures that are not attainable. To prove this we study the geometry of projections onto a finite number of Fourier coefficients, and fully characterize a neighbourhood of a certain symmetrized delta measure. Outside this neighbourhood, the attainable measure can have quite complicated "fractal" singularities. Our results imply that these arise from prime powers - singularities do not appear for circles of radius $\sqrt{n}$ when n is square free.

Exponential sums associated with approximately translation-invariant systems

Trevor Wooley
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School of Mathematics, University of Bristol

The so-called "efficient congruencing method" delivers strong estimates for mean values associated with translation-invariant systems of Diophantine equations, as is the case in Vinogradov's mean value theorem. Indeed, in many respects these estimates come close to achieving, or even furnish, best possible conclusions. In this talk we examine the extent to which such estimates may be obtained also when the exponential sums are associated with incomplete translation-invariant systems of Diophantine equations. If time permits we will describe connections with recent work of Bourgain and Demeter on l^2 decoupling conjectures, this being associated with still more general exponential sums.